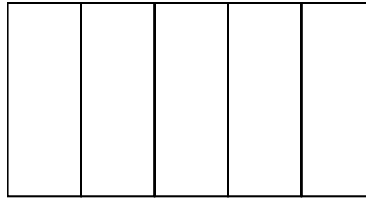


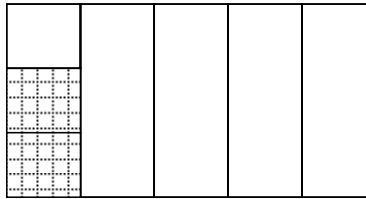
Multiplying Fractions and Mixed Numbers - Green Problems

The diagram below shows a rectangle divided into fifths.



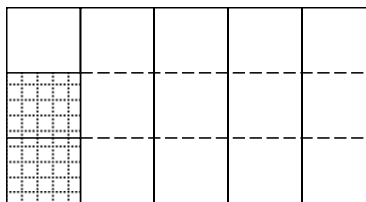
The large rectangle is divided into five equal parts. Each part is $\frac{1}{5}$ of the largest rectangle.

The diagram below shows the rectangle with $\frac{2}{3}$ of $\frac{1}{5}$ of the large rectangle shaded.



$\frac{1}{5}$ of the large rectangle is divided into three equal parts,
with two of the three parts shaded.

The diagram below shows $\frac{2}{3}$ of $\frac{1}{5}$ of the large rectangle is equal to $\frac{2}{15}$ of the rectangle.



By extending the thirds all the way across the width of the large rectangle, we see the large rectangle is divided into fifteen equal parts. Two of the fifteen parts are shaded.

In mathematics, the word "of" often indicates multiplication.

The above diagrams show $\frac{2}{3}$ of $\frac{1}{5} = \frac{2}{15}$. Therefore, we see $\frac{2}{3} \cdot \frac{1}{5} = \frac{2 \cdot 1}{3 \cdot 5}$
 $= \frac{2}{15}$

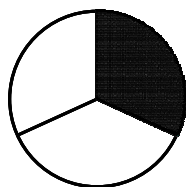
By analyzing the above example we can see the rule for multiplying two fractions.

Multiply numerator by numerator and denominator by denominator

Solution

$$\frac{1}{2} \bullet \frac{1}{3} = \frac{1\bullet 1}{2\bullet 3}$$
$$= \frac{1}{6}$$

Let's look at one of our fraction models to see if $\frac{1}{2}$ of $\frac{1}{3}$ equals $\frac{1}{6}$.



A circle divided into 8 equal sectors, with 1 sector shaded.

We see that $\frac{1}{2}$ of $\frac{1}{3}$ does equal $\frac{1}{6}$. The answer we got using our rule does make sense.

First rewrite the whole number or mixed number as an improper fraction.

$$\frac{5}{6} \cdot 7\frac{1}{2} = \frac{5}{6} \cdot \frac{15}{2}$$

$$\begin{aligned}\frac{2}{3} \text{ of } 27 &= \frac{2}{3} \bullet 27 \\ &= \frac{2}{3} \bullet \frac{27}{1} \\ &= \frac{2 \bullet 27}{3 \bullet 1} \\ &= 18\end{aligned}$$

$$= 6\frac{1}{4}$$

Notice in *Example 2b*, after multiplying the two fractions and getting $\frac{75}{12}$, $\frac{75}{12}$ was simplified by dividing both the numerator and denominator by their common factor, 3.

Common factors of the numerator and denominator of either of the two fractions being multiplied can be "cancelled" before multiplying as well.

<p>Example 3 Multiply $\frac{3}{10} \cdot \frac{6}{7}$</p> <p><i>Solution:</i> $\frac{3}{10} \cdot \frac{6}{7} = \frac{3}{\cancel{10}^5} \cdot \frac{\overset{3}{\cancel{6}}}{7}$ by dividing the 6 and the 10 by 2</p> <p style="margin-left: 150px;"> $= \frac{3 \cdot 3}{5 \cdot 7}$ $= \frac{9}{35}$ </p>

Exercise Set

1. What is the rule for multiplying two fractions?
2. What must you do first in order to multiply a whole number or a mixed number by a fraction?
3. Show that the solution to *Example 1* makes sense.
4.
 - a. In mathematics, what does the word "of" often indicate?
 - b. How do you find $\frac{2}{3}$ of 27?
5.
 - a. When evaluating the product of two fractions, does it make a difference whether you cancel common factors from the numerator and denominator before or after multiplying?
 - b. Evaluate the product in *Example 3*, without cancelling before multiplying? Compare your answer in *Example 3*.
6.
 - a. Find $\frac{1}{2}$ of $\frac{3}{4}$
 - b. Draw a diagram showing $\frac{1}{2}$ of $\frac{3}{4}$
7.
 - a. Find $\frac{3}{4}$ of 10
 - b. Draw a diagram showing $\frac{3}{4}$ of 10
8. Mr. Allrich has 162 students in his five math classes. $\frac{2}{3}$ of his students passed their final exam. How many students passed their final exam? How many students failed?
9. $\frac{4}{9}$ of Mr. Allrich's 162 students are in their senior year in high school. How many of his students are in their senior year?
10. Find $\frac{1}{2}$ of $6\frac{1}{2}$. Explain why your answer makes sense.

Evaluate each of the following products. Simplify your answers.

11. $\frac{3}{8} \cdot \frac{2}{3}$

12. $\frac{3}{4} \cdot 14$

13. $\frac{6}{7} \cdot \frac{7}{6}$

14. $1\frac{1}{2} \cdot 1\frac{1}{2}$

15. $\frac{2}{3} \cdot \frac{9}{12}$

16. $10 \cdot \frac{1}{3}$

17. $3\frac{4}{5} \cdot \frac{5}{6}$

18. $\frac{2}{9} \cdot 1\frac{1}{3}$

19. $2\frac{1}{2} \cdot \frac{1}{4}$

20. $2\frac{1}{10} \cdot 1\frac{1}{4}$

21. $\frac{3}{8} \cdot \frac{4}{3} \cdot \frac{2}{4}$

22. $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}$

23. The **reciprocal** of a fraction is found by switching the numerator and the denominator of the fraction. For example, the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$. Determine the reciprocal of each of the following numbers.

a. $\frac{2}{3}$

b. $\frac{3}{10}$

c. $\frac{3}{2}$

d. $5\frac{1}{2}$ (first convert to an improper fraction)

e. 4

24. Determine each of the following products of fractions and their reciprocals. Simplify your answers.

a. $\frac{2}{9} \cdot \frac{9}{2}$

b. $\frac{2}{1} \cdot \frac{1}{2}$

c. $\frac{5}{2} \cdot \frac{2}{5}$

d. $6\frac{2}{3} \cdot \frac{3}{20}$

e. What do you notice about the products of fractions and their reciprocals?

25. Rochelle ate $\frac{1}{4}$ of a pizza. Walt and Rollin evenly split the rest of the pizza. What fraction of the whole pizza did Walt and Rollin each get?

26. There were $4\frac{1}{2}$ gallons of gasoline in Booker's gas can. He used $\frac{1}{3}$ of the gasoline in the can to fill up the gas tank on his lawnmower.

a. How many gallons of gasoline did Booker pour into his lawnmower?

b. How many gallons of gasoline remained in his gas can?

27. One day, Sareth baked a cake. When it was done, he ate half of it and left the rest on the kitchen table. His brother Sothy came by and ate half of what he found and left the rest behind. Their sister Soklon came by later and ate half of what she found and left the rest. What fraction of the whole cake was left after Soklon came by?

28. Juanita is planning to bake cookies to bring to class. She needs to double the recipe to make enough cookies for everyone. Determine how much of each ingredient Juanita

needs in order to bake cookies for her class.

$\frac{1}{2}$ cup butter

1 teaspoon vanilla

$\frac{3}{4}$ cup brown sugar

$1\frac{1}{4}$ cups flour

2 eggs

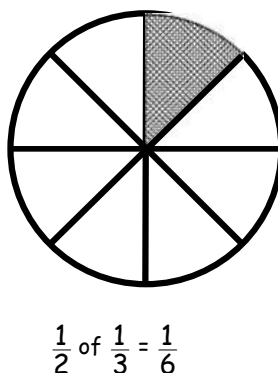
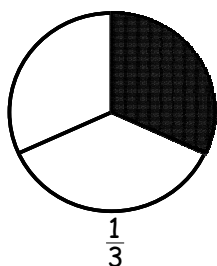
$\frac{2}{3}$ cup chocolate chips

29. Linh has borrowed Juanita's cookie recipe but he only has one egg. Determine how much of each of the other ingredients Linh will need to bake half of a batch of cookies.

Multiplying Fractions and Mixed Numbers - Green Solutions Part I

Exercise Set

1. Multiply the numerator by the numerator and the denominator by the denominator
2. First you must rewrite the whole or mixed number as an improper fraction.
3. $\frac{1}{2} \bullet \frac{1}{3}$ (Read: $\frac{1}{2}$ of $\frac{1}{3}$)



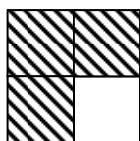
4. a. "of" indicates multiplication.
b. You multiply $\frac{2}{3}$ by $\frac{27}{1}$.

5. a. No it does not.

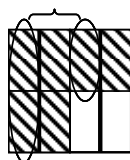
b. $\frac{3}{10} \bullet \frac{6}{7} = \frac{3 \bullet 6}{10 \bullet 7} = \frac{18}{70} = \frac{9}{35}$

The answer is the same as the answer in Example 3.

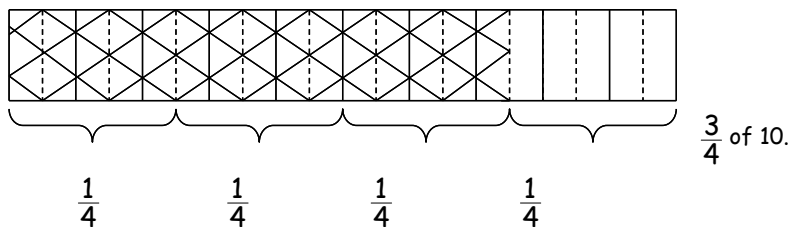
6. $\frac{3}{4}$



$\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$



7. $\frac{15}{2}$ or $7\frac{1}{2}$



8. 108 passed and 54 failed.

9. 72 students are in their senior year.

10. $3\frac{1}{4}$; This makes sense because $\frac{1}{2}$ of 6 is 3 and $\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{4}$.

11. $\frac{1}{4}$

12. $10\frac{1}{2}$

13. 1

14. $2\frac{1}{4}$

15. $\frac{1}{2}$

16. $3\frac{1}{3}$

17. $3\frac{1}{6}$

18. $\frac{8}{27}$

19. $\frac{5}{8}$

20. $2\frac{5}{8}$

21. $\frac{1}{4}$

22. $\frac{1}{4}$

23. a. $\frac{3}{2}$

b. $\frac{10}{3}$

c. $\frac{2}{3}$

d. $\frac{2}{11}$

e. $\frac{1}{4}$

24. a - d all answers are 1.

e. The product of a fraction and its reciprocal is 1.

25. $\frac{3}{8}$

26. a. $1\frac{1}{2}$ gallons b. 3 gallons

27. $\frac{1}{8}$

28. 1 cup of butter
2 teaspoons of vanilla

$1\frac{1}{2}$ cup brown sugar

$2\frac{1}{2}$ cups flour

4 eggs

$1\frac{1}{3}$ cups of chocolate chips

29. $\frac{1}{4}$ cup of butter

$\frac{1}{2}$ teaspoon vanilla

$\frac{3}{8}$ cup brown sugar

$\frac{5}{8}$ cups flour

1 egg

$\frac{1}{3}$ cup chocolate chips